

Reheating, Dark Matter and Baryon Asymmetry: a Triple Coincidence in Inflationary Models

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A scenario in which primeval black holes (PBHs) form at the end of an extended inflationary period is capable of producing, via Hawking radiation, the observed entropy, as well as the observed dark matter density in the form of Planck mass relics. The observed net baryon asymmetry is produced by sphaleron processes in the domain wall surrounding the PBHs as they evaporate around the electroweak transition epoch. The conditions required to satisfy these three observables determines the PBH formation epoch, which can be associated with the end of inflation, at $t \sim 10^{-32}$ s.

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I. INTRODUCTION

The possibility of primeval black hole (PBH) formation in the early Universe has been known for a long time [1, 2]. The realization that quantum effects lead to their evaporation [3] led to investigations of possible constraints on the PBH mass spectrum from their consequences for various astrophysical backgrounds [4, 7] and their possible dynamical role as dark matter [5]. The interplay between PBHs as a source of CDM affecting cosmological dynamics, and their evaporation as a source of entropy and particles [6] affecting nucleosynthesis and CMB observations leads to constraints on their mass spectrum over a wide mass range [8, 9]. Since black hole evaporation manifestly violates T-invariance, CP must be violated as well so that baryon number is not conserved [11], which led to the suggestion, in the context of GUTs, that the baryon asymmetry of the Universe could be thereby explained [12, 13]. In the context of inflationary models, PBH formation can be triggered by large amplitude inhomogeneities caused by bubble nucleation over scales comparable to the horizon, which can collapse into black holes at the end of an extended inflationary period [15].

The evaporation of PBHs with mass less than $\sim 10^{15}$ g, on timescales less than a Hubble time [3], can lead either to complete evaporation, or may stop at a Planck scale mass $m_{Pl} \sim 10^{-5}$ g [16]. Much work has been done since then on the possibility of PBH formation at phase transitions, on the dynamics of their collapse and on their possible role in large scale structure formation, e.g. [17, 18] and references therein.

In what follows, we concentrate on the potential of PBHs formed at the end of extended inflation in providing a mechanism for the production of the current observed entropy per baryon, the inferred dark matter density, and as a possible source for the baryon asymmetry in the Universe. We emphasize a remarkable triple coincidence for the conditions required to produce these three quantities, pointing towards a well defined epoch for PBH formation around $t \sim 10^{-32}$ s, which can be

identified with the end of inflation.

In this model, the absolute entropy of the universe is given by the entropy of a gas of standard model particles at the initial temperature $T_{BH} \sim 300$ GeV, produced by PBHs created at $t \sim 10^{-32}$ s which evaporate at $t_{BH} \sim 10^{-12}$ s, identified with the reheating time. This same PBH evaporation leads also to a dark matter component, assumed to consist of approximately Planck mass remnants with a mass density approximately equal to the PBH density times the Planck mass. The net baryon density and asymmetry is also related to the PBH evaporation, through sphaleron processes in a domain wall structure surrounding the PBHs. The presently observed entropy, dark matter density and net baryon to entropy ratio are obtained for a unique value of the reheating time $t_{BH} \sim 10^{-12}$ s, which coincides with the electroweak timescale, and which defines a unique time for the end of inflation at $t_{end} \sim 10^{-32}$ s.

II. PBHS, REHEATING AND ENTROPY

At the Planck time $t_{Pl} = (\hbar G/c^5)^{1/2} \sim 10^{-43.3}$ s the Planck mass $m_{Pl} = (\hbar/t_{Pl}c^2) \sim (\hbar c^3/G) \sim 10^{-4.7}$ g, corresponding to the Planck energy $E_{Pl} \sim 10^{19.05}$ GeV, is within a particle horizon whose size is the Planck length $\ell_{Pl} \sim ct_{Pl} \sim 10^{-32.83}$ cm. Any PBHs formed before or during inflation would have had their energy density diluted by the exponential expansion of the scale factor to a negligible value, so that PBH formation is of interest mainly after the end of inflation, at $t \gtrsim t_{end}$.

The difficulties of the original inflation model are resolved most simply in models of extended or hyperextended inflation [19], which is generally taken to end around an epoch $t_{end} \sim 10^{-32 \pm 6}$ s, at which time the energy scale has dropped to $E \sim \rho^{1/4} \sim 10^{13 \pm 3}$ GeV. At this time the Universe is cold, due to adiabatic cooling during the expansion of the scale factor by sixty or more e-foldings, so the pressure is essentially zero, and the equation of state is correspondingly soft. Primordial energy density fluctuations coming into the hori-

zon at $t \sim t_{end}$ may be of the canonical inflationary (Harrison-Zeldovich) type, with relative amplitudes $\delta_{end} \equiv (\delta\rho/\rho)_{t_{end}} \sim 10^{-4}$, and/or may be large amplitude fluctuations caused by chaotic conditions associated with bubble nucleation at t_{end} , where one may expect $\delta_{end} \sim 1$, e.g. [15]. The latter fluctuations can cause PBHs to form almost immediately at $t_1 \sim t_{end}$, with a mass M_{BH} which is a fraction $\eta \lesssim 1$ of the mass in the horizon $M_{hor} \sim m_{Pl}(t/t_{Pl})$ at that time,

$$M_{BH}(t_1) \simeq \eta m_{Pl}(t_1/t_{Pl}) \simeq 10^{6.6} \eta t_{1,-32} \text{ g} \quad (1)$$

or $M_{BH} \simeq 10^{30.3} \eta t_{1,-32} \text{ GeV}$, where $t_{1,-32} = (t_1/10^{-32}\text{s})$. The temperature associated with a black hole of mass M_{BH} is

$$\begin{aligned} T_{BH} &= (m_{Pl}^2/8\pi M_{BH}) \\ &= 10^{6.4} M_{6.6}^3 \text{ GeV} = 10^{6.4} \eta^{-1} t_{1,-32} \text{ GeV}. \end{aligned} \quad (2)$$

In the standard treatment, these PBHs evaporate on a timescale

$$\begin{aligned} t_{BH}(M_{BH}) &= g_*^{-1} (M_{BH}/m_{Pl})^3 t_{Pl} \\ &\simeq 10^{-11.4} g_{*,2}^{-1} (\eta t_{1,-32})^3 \text{ s}, \end{aligned} \quad (3)$$

where $g_* = 10^2 g_{*,2} \sim 106.75$ is the number of degrees of freedom in the early universe for the standard model[10]. Most of the evaporated energy goes into radiated photons and particles.

For evaporation times much longer than the epoch of formation, $t_{BH} \gg t_1$, the epoch (age of the Universe), at which PBHs of mass M_{BH} evaporate is $t \sim t_{BH} \sim 10^{38} M_{BH} \text{ s}$. Thus even if the perturbations coming into the horizon at the end of inflation had only the canonical amplitude $\delta_{end} \sim 10^{-4}$, they grow with the scale factor of the Universe a as $\delta \propto a \propto t^{2/3}$ (for a matter dominated [MD] Universe, if the equation of state is cold). The collapse time t_{col} at which the fluctuations achieve large amplitude $\delta \sim 1$ is much smaller than t_{BH} , and for fluid-like perturbations the epoch at which PBHs of mass M_{BH} evaporate is again $t \sim t_{BH}$.

If $\beta(M_{BH})$ is the fraction of the energy density of the Universe which collapses into PBHs of mass $\sim M_{BH}$ at the epoch t_1 , the radiation produced by the PBHs at the evaporation time t_{BH} , after having relaxed with the environment, results in a specific entropy per baryon $S = s/n_B$ of $S \simeq (1 + S_i)\beta(M)(M/m_{Pl})$ [9], where S_i is the initial entropy per baryon before PBH evaporation, assuming $\beta \ll 1$. This can be used to set constraints on the fraction β of PBHs of mass M , and can also contribute to producing some or possibly most of the entropy of the universe. Generalizing this argument to an inflationary scenario [15], with $S_i \simeq 0$ as expected from adiabatic cooling at $t_1 \sim t_{end}$, assuming that the PBH mass is a fraction ηM_{Hor} of the mass in the horizon at the collapse time t_1 , the initial energy density in a PBH component is

$$\rho_{BH}(t_1) = (3/32\pi)\beta(m_{Pl}/t_1^3)(t_1/t_{Pl}), \quad (4)$$

while the remaining fraction $(1 - \beta)$ goes into relativistic particles or radiation, $\rho_R(t_1)$. For plausible values of $\eta \lesssim 1$, $10^{-10} \lesssim \beta \lesssim 1$, the universe expansion is initially dominated by radiation, $a \propto t^{1/2}$, but after a time $t_2 = [(1 - \beta)^{1/\beta^2}]t_1$ it becomes PBH dominated [15], $a \propto t^{2/3}$. The PBHs evaporate at $t \simeq t_{BH} \gg t_1$, injecting into the universe a radiation energy density $\rho_R(t_{BH}) = (3/32\pi)(1 - \beta)\eta^{-6}g_*^2(t_{Pl}/t_1)^3(m_{Pl}/t_1^3)$, which can be re-expressed as a function of the initial mass of the PBHs which evaporate at t_{BH} ,

$$\rho_R(t_{BH}) = (3/32\pi)(1 - \beta)(m_{Pl}/t_{Pl}^3)(m_{Pl}/M_{BH})^2. \quad (5)$$

This newly injected radiation component is much larger than the diluted radiation produced at time t_1 , and it provides henceforth the dominant energy form in the universe, which again expands as $a \propto t^{1/2}$ (until t_{eq}). At this time the universe acquires an entropy density $s(t_{BH}) = (2\pi^2/45)g_*T(t_{BH})^3$, being reheated to a temperature $T(t_{BH})$ given by

$$\begin{aligned} T(t_{BH}) &= (30/g_*\pi^2)^{1/4} \rho_R(t_{BH})^{1/4} \\ &= \left(\frac{90g_*[1 - \beta]}{32\pi^3}\right)^{1/4} \left(\frac{m_{Pl}}{t_{Pl}^3}\right)^{1/4} \left(\frac{m_{Pl}}{M_{BH}}\right)^{3/2} \\ &\simeq 250g_{*,2}^{1/4}(1 - \beta)^{1/4} M_{6.6}^{-3/2} \text{ GeV} \end{aligned} \quad (6)$$

Taking the standard value for the matter-radiation equilibrium epoch $a_{eq} = 4.3 \times 10^{-5}(\Omega_{M,0}h^2)^{-1}$, the PBH evaporation epoch corresponds to $a_{BH} = 4.1 \times 10^{-16} M_{6.6}^{3/2}$, and from the entropy scaling $T^3 a^3 g_* = \text{constant}$ with $g_{*,BH} \simeq 106$ and $g_{*,0} = 3.9$ one obtains a present day radiation temperature $T_0 \simeq 3.0 \times 10^{-4}(1 - \beta)^{1/4} \text{ eV}$, close to the observed value of $2.5 \times 10^{-4} \text{ eV}$ (See Fig. 1). Notice that if one were only trying to explain the current entropy or the current radiation temperature, one could in principle also satisfy this with, e.g. earlier evaporation times or higher T_{BH} values. However, if in addition one demands that the PBH evaporation should also lead to the currently observed dark matter density, the evaporation time becomes determined, as discussed below.

III. DARK MATTER

The evaporation timescale (3) remains approximately the same even if mass loss stops after the PBH has shrunk down to a Planck mass $\sim 10^{-5} \text{ g}$, since by this time it will have radiated away most of its energy in the form of photons and particles. The semi-classical quantum evaporation treatment breaks down near $\sim m_{Pl}$, requiring taking quantum gravity effects into account, and several authors have argued that the process leaves behind stable relics of approximately a Planck mass [16]. These would behave as non-relativistic matter, and in order not to exceed limits on the current dark matter density, they imply constraints on the epoch at which they evaporated, and therefore also on the epoch at which they formed

[15, 18]. At the epoch t_{BH} when they evaporate, if each PBH leaves a relic of mass κm_{Pl} , the relic matter density ρ_M is

$$\rho_M = (3/32\pi)(1 - \beta)g_*^2(m_{Pl}/M_{BH})^7(\kappa m_{Pl}/t_{Pl}^3) \quad (7)$$

At this time t_{BH} the PBH-contributed radiation density (5) is dominant, and the ratio of radiation (including relativistic particles) density to relic (dark) matter density is

$$(\rho_R/\rho_M)_{t_{BH}} \simeq (M_{BH}/\kappa m_{Pl}) \equiv (\eta t_1/\kappa t_{Pl}), \quad (8)$$

which is $\simeq 2 \times 10^{11} \kappa^{-1} M_{6.6} = 2 \times 10^{11} \kappa^{-1} \eta t_{1,-32}$. This ratio decreases as a^{-1} , and at t_{eq} its value is

$$\begin{aligned} (\rho_R/\rho_M)_{t_{eq}} &\simeq 2M_{6.6}^{5/2} \kappa^{-1} \Omega_{M,0} h^2 \\ &\simeq 2t_{1,-32}^{5/2} \eta \kappa^{-1} \Omega_{M,0} h^2, \end{aligned} \quad (9)$$

close to unity, as needed for dark matter. The evaporation at $t_{BH} \sim 10^{-11.4}$ s of PBHs formed at $t \sim 10^{-32}$ s leads therefore to a plausible model for explaining the reheating of the Universe after the end of inflation, leading to the right amount of present day entropy, as well as providing a source for the present day dark matter density. The latter could be in the form of stable Planck mass relics, or possibly stable, weakly interacting decay products of such relics, with the same total mass. This is achieved if (i) the end of inflation occurs at $t_{end} \sim 10^{-32}$ s, (ii) PBHs collapse is dominated by fluctuations coming into the horizon at $t_1 \sim t_{end}$, aided by the soft equation after the end of inflation, and iii) the fraction of the energy density of the Universe collapsing into PBHs at that epoch is $10^{-10} \lesssim \beta \lesssim 1$ (see previous section). This range is mostly unconstrained by current observational restrictions on PBH mass spectra [18]. The choice of $t_1 \sim 10^{-32}$ s is then essentially determined, if we want to explain both the current entropy and the current dark matter.

IV. BARYON ASYMMETRY

It is generally thought that the baryon asymmetry must have been generated by the epoch at which the Universe cooled below the electroweak energy scale $T_{ew} \sim 300 T_{ew,300} \text{ GeV}$ [13, 15]. This corresponds, extrapolating back from the present epoch, to a scale factor $a_{ew} \sim 2.6 \times 10^{-16} T_{ew,300}^{-1}$ and an epoch $t_{ew} \sim 10^{-11.9} (\Omega_{M,0} h^2)^{-2} T_{ew,300}^{-2}$ s. This electroweak transition epoch t_{ew} is, numerically, essentially the same as the evaporation epoch $t_{BH} \sim 10^{-11.4}$ s defined in equation (3), for PBHs of mass $M_{BH} \sim 10^{6.6}$ g [eq. (1)] which formed at the epoch $t_1 \simeq 10^{-32}$ s.

The coincidence between the PBH evaporation timescale t_{BH} and the electroweak timescale t_{ew} is remarkable. The first is derived from a requirement to explain the reheating and the observed entropy/DM ratio starting from an inflationary early universe scenario,

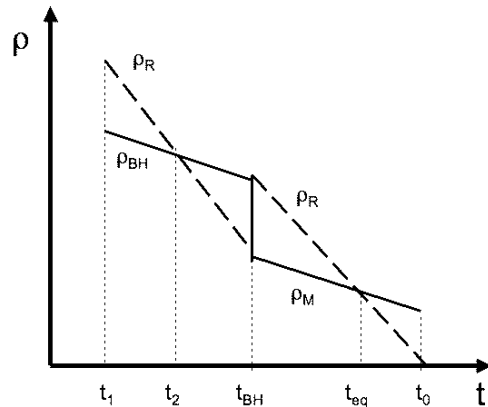


FIG. 1: The radiation, primordial black hole and subsequent dark matter evolution as a function of cosmological time.

while the second is determined from a particle physics energy scale and the more recent dynamics of the Universe. The fact that the Universe is baryon asymmetric provides, in fact, an additional constraint on the epoch t_{BH} at which PBHs evaporate, if the baryon asymmetry arises from baryonic decay products of PBH evaporation, which is manifestly CP-violating process.

The requirement that any net baryon number $n_B = n_b - n_{\bar{b}}$ produced in PBH evaporation should not be washed out by CP-violating currents expected at the electroweak transition epoch t_{ew} imposes the requirement $t_{BH} \gtrsim t_{ew}$. This additional requirement is in fact satisfied by the entropy to dark matter constraint (9), requiring $t_{BH} \sim 10^{-11.4}$ s, which in turn constrains, through equation (1,3), the horizon entrance time of the PBH perturbations to be $t_1 \simeq 10^{32}$ s. In addition, in order for the energy density of these PBHs not to be diluted to a negligible value, they must be born after the end of inflation, $t_1 \gtrsim t_{end}$. The double constraint on $t_1 \simeq 10^{-32}$ s from the reheating/entropy/DM requirement on the one hand, and on $t_2 \gtrsim t_{ew}$ to preserve any baryon asymmetry on the other hand, in turn constrains the end of inflation to occur at $t_{end} \simeq t_1 \simeq 10^{-32}$ s. In fact, either one of the previous two constraints acting individually (as long as PBHs are responsible either for the entropy/dark matter ratio or the baryon asymmetry) is enough to require $t_{end} \simeq 10^{-32}$ s. The fact that both independent constraints acting simultaneously require the same value of t_{end} is again remarkable.

The baryogenesis mechanism of [14] included mechanisms occurring at GUT temperatures, assuming that PBHs with $T_{BH} > 10^{14}$ GeV radiate bosons which decay into a net baryon number. (A different PBH baryogenesis mechanism in ekpyrotic/cyclic models [22] has been discussed by [23]). However, any GUT scale baryon asymmetry can be washed out by Sphaleron processes during the electroweak phase transition. Sphalerons are

non-trivial topological field configurations which generate a net $B - L$ number. It was shown by Cohen, Kaplan and Nelson [21] that it is possible to use the electroweak sphaleron (instantons) to generate baryon asymmetry through the well known ABJ anomaly equation:

$$\partial_\mu J_B^\mu = N_f \left(\frac{g^2}{32\pi^2} W\tilde{W} - \frac{g'^2}{32\pi^2} B\tilde{B} \right) \quad (10)$$

where N_f is the number of families, $W_{\mu\nu}$ is the weak field strength, $B_{\mu\nu}$ is the hypercharge field strength and g and g' are the gauge couplings. The CKN mechanism states that baryogenesis can be spontaneous in the sense that a derivative coupling between a scalar field and the baryon number current is induced in general:

$$\mathcal{L}_{\text{ind}} = \partial_\mu \phi N_f \left[\frac{g^2}{32\pi^2} Y_{\text{CS}}(\text{A}_{\text{SU}(2)}) + \frac{g'^2}{32\pi^2} Y_{\text{CS}}(\text{A}_{\text{U}(1)_Y}) \right] \quad (11)$$

where in general $F(A) \wedge F(A) = dY_{\text{CS}}(A)$, and substituting from eq (10) we get

$$\mathcal{L}_{\text{ind}} \simeq \partial_\mu \phi J^\mu \quad (12)$$

However the coincidence of PBH formation during and before the electroweak phase transition temperature gives us a clue as to the origin of this field ϕ . If the field ϕ is associated with the phase of the Higgs field we may be able to naturally generate the baryon asymmetry. Indeed such a mechanism was made concrete by Nagatani [24]. In this mechanism the Higgs field forms a spherical domain wall around the PBH due to spontaneous electroweak symmetry breaking. The gradient in the domain wall is the CP violating phase which also acts as the chemical potential to generate the net baryon asymmetry due to sphaleron processes. The domain wall configuration is expressed as:

$$\langle \phi_1^0(r) \rangle = \begin{cases} 0 & (r \leq r_{\text{DW}}) \\ v_1 f(r) e^{-i\Delta\theta(1-f(r))} & (r > r_{\text{DW}}) \end{cases} \quad (13)$$

where $f(r) = \sqrt{1 - (T(r)/T_{\text{weak}})^2}$, $T(r)$ is the local temperature measured at a radius r from the black hole center. This temperature gradient is determined by the radiation energy density gradient produced by the radiation outflow from the black hole.

In this configuration of the Higgs vacuum expectation value, the width of the domain wall d_{DW} is equal to the depth of the symmetric region. The Hawking radiation (particles) emanating from the black hole traverse this domain wall, and the energy gradient in the wall induces a sphaleron transition which creates a net baryon number from the Hawking radiation. The net baryon number n_B resulting from this process can be calculated directly [24], and for PBH temperatures $T_{\text{BH}} \sim 10^{6.5} - 10^{7.5} \text{ GeV}$ the resulting net baryon number, and the ratio of the net baryon to entropy (where the latter is as calculated in the previous section) is $n_B/s \simeq 10^{-10}$, satisfying the BBN constraints. Remarkably, as shown in the previous two sections, this temperature is essentially the same temperature (2) corresponding to PBHs formed at $t_1 \sim 10^{-32} \text{ s}$

and evaporating at $t_{\text{BH}} \sim 4 \times 10^{-12} \text{ s} \sim t_{\text{ew}}$, which can produce both the observed entropy and the observed dark matter density. These same PBHs can therefore also produce the right net baryon number and the entropy per baryon of the universe.

V. DISCUSSION

The possibility that three major observational parameters of the universe, namely the entropy density, the dark matter density and the net baryon to entropy ratio, may be simultaneously explained by a single mechanism is remarkable. In this scenario the reheating and the entropy is produced by the evaporation of primordial black holes. If, as has been widely surmised, these leave relics whose mass is of order the Planck mass per evaporating black hole, these can provide the dark matter density. For one or both of the above to come out right, the PBH mass must be in the kiloton range. A newer element, in addition to the above, is that the difference between the PBH temperature and the temperature of the universe provides a temperature gradient, through which evaporating particles can undergo CP-violating transitions leading to a net baryon number. A specific PBH evaporation domain wall mechanism can give the observed net baryon number and baryon to entropy ratio, in agreement with BBN constraints, when the PBH temperature is in the PeV range, corresponding again to the kiloton mass range.

The entropy, by itself, could in principle be produced by a range of PBH collapse times $t_1 \lesssim 10^{-32} \text{ s}$. However, the additional requirement of relating also the dark matter density, the net baryon number, or both, to the evaporation process narrows the PBH formation time to the epoch $t_1 \sim 10^{-32} \text{ s}$. Since PBHs with a significant energy density must arise at or after this epoch, this can be identified with the end of inflation. This same epoch occurs independently in hyperextended models of inflation where the non-minimal coupling is something other than quadratic in the scalar field, leading also to a prediction [25] of a gravitational wave background.

The PBH formation epoch $t_1 \sim t_{\text{end}} \sim 10^{-32} \text{ s}$ also determines the reheating temperature $T \sim 300 \text{ GeV}$, caused by the PBH evaporation at the epoch $t_{\text{BH}} \sim 10^{-12} \text{ s} \sim t_{\text{ew}}$. The triple coincidence discussed here, based on the physics of PBH evaporation, provides a strong incentive for identifying PBHs as responsible for three of the key parameters of cosmological models, namely the current entropy, the dark matter, and the net baryon asymmetry with the right baryon to entropy ratio. It also provides an upper limit for the end of inflation at the epoch $t_{\text{end}} \sim 10^{-32} \text{ s}$.

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